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Effect of porosity on natural convective heat transfer in a fluid saturated porous medium

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Abstract

The effect of porosity on natural convective flow and heat transfer in a fluid saturated porous medium has been investigated using Galerkin's finite element method. A generalised non-Darcy flow model with porosity as a separate parameter is used. Results indicate that the non-Darcy regime is highly sensitive to porosity changes. A variation of the order of 40% in the average Nusselt number is possible with change in the porosity, for the Rayleigh and Darcy number ranges considered.

Notation

- c_p specific heat
- Da Darcy number (κ/L^2)
- g acceleration due to gravity
- h heat transfer coefficient
- J viscosity ratio (μ_{eff}/μ_f)
- k_m effective thermal conductivity ($\epsilon k_f + (1-\epsilon)k_s$)
- L side length of the square cavity
- Nu Nusselt number ($h\,L/k_m) = (1/L)\int_0^L(\partial T/\partial x)\mathrm{d}x$
- p Pressure
- Pr Prandtl number (ν_f/α_m)
- Ra Rayleigh number ($g\beta\Delta TL^3/\nu_f\alpha_m$)
- Ra* Darcy–Rayleigh number (Ra Da)
- t time
- T temperature
- u,v velocity components in x and y directions
- x,y coordinate axes

Greek

- α_m effective thermal diffusivity ($k_m/(\rho c_p)_f$)
- ϵ porosity
- β coefficient of thermal expansion
- κ permeability
- μ dynamic viscosity
- ν kinematic viscosity
- ψ stream function
- ρ density
- σ ratio of heat capacities ($\epsilon(\rho c_p)_f + (1-\epsilon)(\rho c_p)_s)/(\rho c_p)_f$

Subscripts

- f fluid
- m porous medium
- s solid

1. Introduction

The application of the porous medium approach to model real life problems such as alloy solidification, flow through turbo machines, flow through the tissues of human body, heat and mass transfer in packed beds, etc., has become an active area of research in recent times. In these situations, the actual flow problem may be approximated with reasonable accuracy, by employing suitable relations for the equivalent medium porosity and flow resistance (Shyy and Vu, 1993; Sinha et al., 1992). In the past, the dependence of porosity has been considered for calculating the effective physical and transport properties of the porous medium and to determine the wall effects near solid boundaries. Very few works in the literature consider porosity as an independent parameter, as far as natural convective flow in a porous medium is concerned. For instance, Lauriat and Prasad (1989) considered the non-Darcy flow effects in a porous medium. Lage (1993), adopting a generalised porous medium approach, developed a correlation for Nusselt number from an approximate scale analysis. Vafai and Tien (1981) and Hsu and Cheng (1990) derived a generalised model using volume averaging technique for forced convection in porous media. The available literature on non-Darcy flow models does not highlight the influence of porosity upon flow and transport. In the present work, the effects of porosity on natural convective flow and heat transfer are analysed in detail, in the non-Darcy flow regime.

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2. Problem formulation and governing equations

The problem considered is a two-dimensional square cavity filled with fluid saturated porous medium. The vertical walls are maintained at different temperatures and an adiabatic condition is imposed on the horizontal sides. All the properties except density in the buoyancy term are considered to be constant. The Prandtl number is assumed to be unity in all the computations for the sake of simplicity. Invoking the Boussinesq approximation, the non-dimensional governing equations (Nithiarasu et al., 1997) are given by:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

x-momentum equation:

$$\frac{1}{\epsilon}\frac{\partial u}{\partial t}+\frac{1}{\epsilon^2}u\frac{\partial u}{\partial x}+\frac{1}{\epsilon^2}v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\mathrm{Pr}}{\epsilon}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\mathrm{Pr}}{\mathrm{Da}}Ju-\frac{1.75}{\sqrt{150}}\frac{\sqrt{u^2+v^2}}{\sqrt{\mathrm{Da}}}\frac{u}{\epsilon^{3/2}}, \tag{2}$$

y-momentum equation:

$$\frac{1}{\epsilon}\frac{\partial v}{\partial t}+\frac{1}{\epsilon^2}u\frac{\partial v}{\partial x}+\frac{1}{\epsilon^2}v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\mathrm{Pr}}{\epsilon}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\mathrm{Pr}}{\mathrm{Da}}Jv-\frac{1.75}{\sqrt{150}}\frac{\sqrt{u^2+v^2}}{\sqrt{\mathrm{Da}}}\frac{v}{\epsilon^{3/2}}+\mathrm{Ra}\,\mathrm{Pr}\,T. \tag{3}$$

Energy equation:

$$\sigma\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The detailed derivation of the above equations is discussed elsewhere (Nithiarasu et al., 1997). These equations are derived from the first principles of conservation of mass, momentum and energy over a fluid saturated porous control volume. In particular, the momentum equations include the effects of fluid inertia, viscous forces, linear and non-linear matrix drag and buoyancy.

Galerkin's finite element method coupled with the Eulerian velocity correction procedure is used in the present study. The Adams–Bashforth scheme is employed for the advection terms and Euler's implicit procedure is adopted for diffusion terms. More details about the finite element formulation are given in Nithiarasu et al. (1996).

3. Results and discussion

The present model is validated (Fig. 1) with the experimental results of Prasad et al. (1985). The geometry considered is an axisymmetric cavity with a radius ratio of 5.333 and an aspect ratio of 1.0. The present predictions are seen to agree well with the experimental Nusselt number values.

In Fig. 2 the flow patterns for two different porosities are superimposed at a Darcy number of 10^{-2} and Rayleigh number of 10^4. It is observed that the vortex strength increases as the porosity of the medium is increased. This increase in vortex strength leads to a thinner velocity boundary layer and steeper velocity as well as temperature gradients. At lower darcy numbers ($\mathrm{Da} < 10^{-6}$), the porosity effect has been observed to be small (not shown in figure).

Table 1 shows the flow parameters at two different Darcy numbers. At $\mathrm{Da}=10^{-6}$ (Darcy regime), change in porosity hardly affects the maximum vertical velocity or the stream function. On the other hand, at $\mathrm{Da}=10^{-2}$ (non-Darcy regime), the maximum vertical velocity and stream function vary by 30% between the cases of $\epsilon=0.4$ and 0.8.

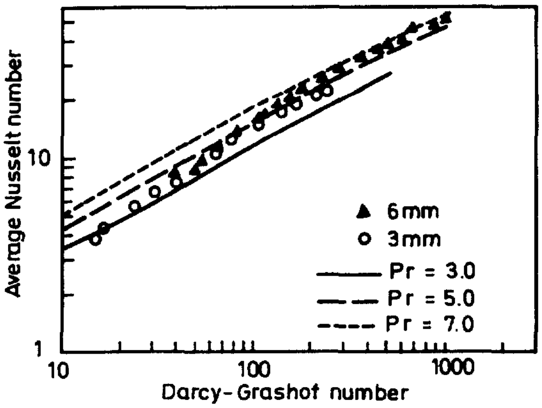

Fig. 1. Comparison of present predictions with the experimental results of Prasad et al. (1985). Glass–water, $3 < \mathrm{Pr} < 5$; $\mathrm{Da}=10^{-6}$; $\epsilon=0.4$; $R^*=5.333$; AR = 1.

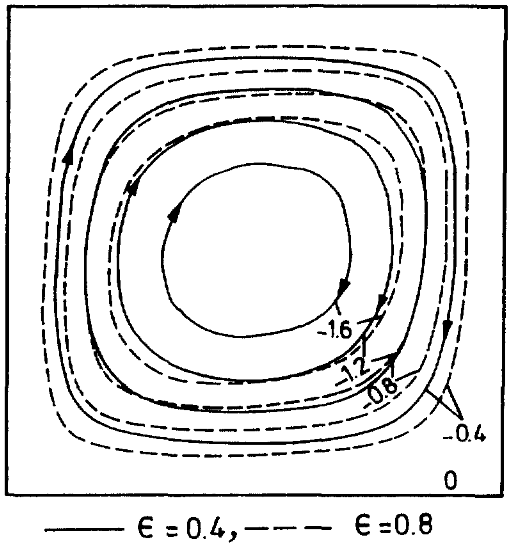

Fig. 2. Effect of porosity on stream line pattern. $\mathrm{Da}=10^{-2}$; $\mathrm{Ra}=10^4$.

Table 1
Flow parameters for different porosities

S. no.	Da	Ra	$\epsilon=0.4$		$\epsilon=0.8$	
			v_{max}	$\lvert\psi_{max}\rvert$	v_{max}	$\lvert\psi_{max}\rvert$
1	10^{-6}	10^8	45.2	4.6	47.7	4.64
3	10^{-2}	10^4	6.9	1.99	9.95	2.65

The average Nusselt number values at different Darcy and Rayleigh numbers are shown in Fig. 3. At lower Darcy–Rayleigh numbers ($\mathrm{Ra}^*=10$) and $\mathrm{Da}=10^{-6}$, no difference is observed between the curves corresponding to the three porosities considered. As the Rayleigh number is increased to higher values, the differences between the curves are perceptible. Therefore, even at lower Darcy numbers, the variation of average Nusselt number with porosity is not negligible at higher Rayleigh numbers.

As Darcy number is increased, in Fig. 3(b) the Ra* limit for significant deviation between the Nusselt number curves corresponding to different porosities decreases. A maximum

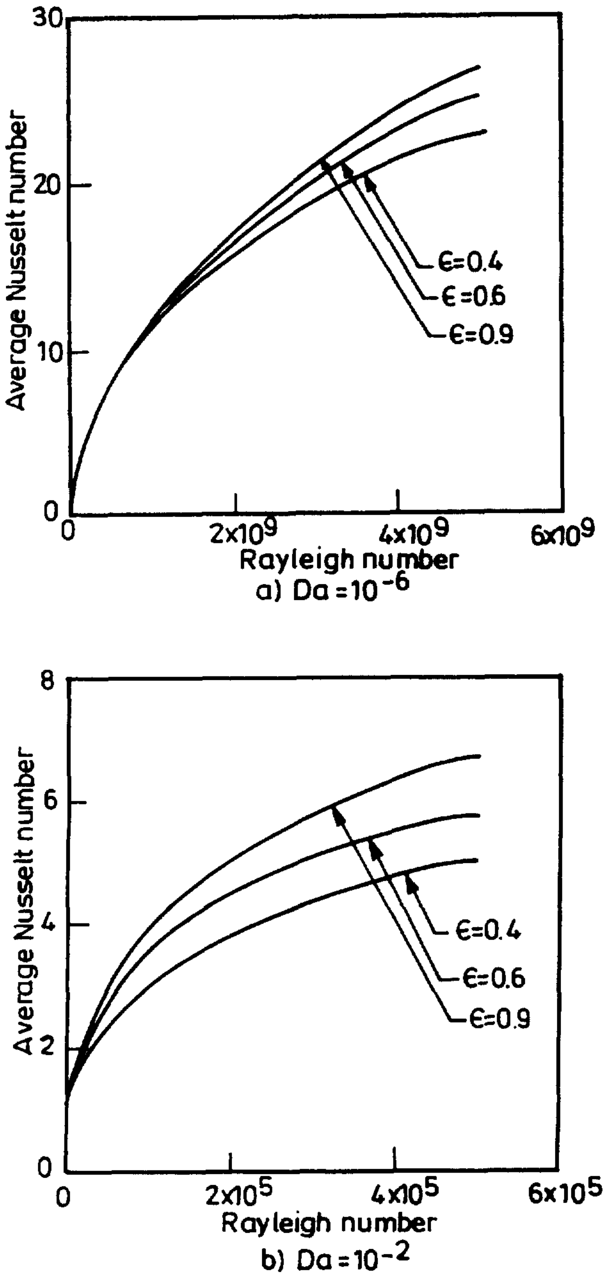

Fig. 3. Average Nusselt number variation with Rayleigh number for different porosity values.

difference of 40% in average Nusselt number between different porosities was observed, at the highest Darcy–Rayleigh number of 5000 considered in the present study.

4. Conclusions

The influence of porosity on natural convective flow and heat transfer has been investigated using a generalised porous medium model. Results show that the porosity significantly affects the flow and heat transfer in a fluid saturated porous medium at higher Darcy numbers. It is shown that the porosity should be considered as an independent parameter in such cases. At low Darcy numbers, the effect of porosity is less but not negligible.

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